Dot Product

Wednesday, May 10, 2023 8:56 AM

review: given vector $\langle v_1, v_2, v_3 \rangle$, the unit vector in direction $\vec{v} \longrightarrow \underbrace{\frac{1}{|v|}}_{\text{length}} \cdot \vec{v}$ } vector of length 1 $= \frac{\vec{v}}{|v|}$

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dot product: $\vec{u} \cdot \vec{v}$ of 2 vectors $\vec{u}$ & $\vec{v}$ is the # ✱ always a # / never vector ✱

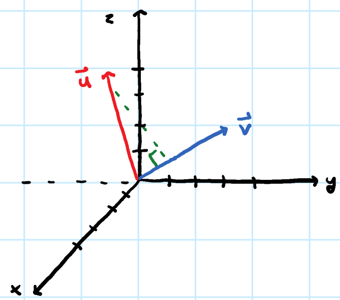
$$\vec{u} \cdot \vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + u_3 \cdot v_3$$

where $\vec{u} = \langle u_1, u_2, u_3 \rangle$, $\vec{v} = \langle v_1, v_2, v_3 \rangle$

ex 1) $\vec{u} = \langle 2, 1, 4 \rangle$, $\vec{v} = \langle -1, 3, 2 \rangle$

then $\vec{u} \cdot \vec{v} = (2 \cdot -1) + (1 \cdot 3) + (4 \cdot 2)$

$= -2 + 3 + 8 = \boxed{9}$

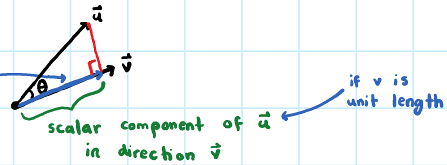


dot product for projecting:

- scalar component of $\vec{u}$ in direction $\vec{v}$ is

  "length"

  $\vec{u} \cdot \frac{\vec{v}}{|v|} \leftarrow$ #



- projection of $\vec{u}$ to $\vec{v}$ is:

  $$\underbrace{\left(\vec{u} \cdot \frac{\vec{v}}{|v|}\right)}_{\#} \cdot \underbrace{\frac{\vec{v}}{|v|}}_{\text{vector}}$$

ex 2) $\vec{u} = \langle 2, 1, 4 \rangle$, $\vec{v} = \langle -1, 3, 2 \rangle$

find projection of u in direction $\vec{v}$

solution: first, who is $\frac{\vec{v}}{|v|}$?

$|v| = \sqrt{-1^2 + 3^2 + 2^2}$

$= \sqrt{14}$

$\frac{\vec{v}}{|v|} = \left\langle \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right\rangle$

then $\vec{u} \cdot \frac{\vec{v}}{|v|} = \left(2 \cdot \frac{-1}{\sqrt{14}}\right) + \left(1 \cdot \frac{3}{\sqrt{14}}\right) + \left(4 \cdot \frac{2}{\sqrt{14}}\right) = \frac{9}{\sqrt{14}}$ (scalar component)

for $\text{proj}_{\vec{v}}(\vec{u})$ we have ...

$\text{proj}_{\vec{v}}(u) = \frac{9}{\sqrt{14}} \cdot \left\langle \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right\rangle$ (vector in direction $\vec{v}$)

$= \boxed{\left\langle \frac{-9}{14}, \frac{27}{14}, \frac{18}{14} \right\rangle}$

or

$= \boxed{\frac{9}{\sqrt{14}} \cdot \frac{1}{\sqrt{14}} \langle -1, 3, 2 \rangle = \frac{9}{14} \vec{v}}$

or

$$= \boxed{\frac{9}{\sqrt{14}} \cdot \frac{1}{\sqrt{14}} \langle -1, 3, 2 \rangle = \frac{9}{14} \vec{v}}$$